

Can 'Unsharp Objectification' Solve the Quantum Measurement Problem?

Paul Busch¹

Received July 4, 1997

The quantum measurement problem is formulated in the form of an insolubility theorem that states the impossibility of obtaining, for all available object preparations, a mixture of states of the compound object and apparatus system that would represent definite pointer positions. A proof is given that comprises arbitrary object observables, whether sharp or unsharp, and besides sharp pointer observables a certain class of unsharp pointers, namely, those allowing for the property of *pointer value definiteness*. A recent result of H. Stein is applied to allow for the possibility that a given measurement may not be applicable to all possible object states, but only to a subset of them. The question is raised whether the statement of the insolubility theorem remains true for genuinely unsharp observables. This gives rise to a precise notion of *unsharp objectification*.

1. INTRODUCTION

The claim of the insolubility of the quantum measurement problem has been given a precise formulation in a series of papers aiming at increasing generality of the premises (see, e.g., Wigner, 1963; d'Espagnat, 1966; Fine, 1970; Shimony, 1974; Brown, 1986). The most recent step provided an extension of the insolubility proof to include measurements of arbitrary sharp or unsharp object observables (Busch and Shimony, 1996). In the present contribution I consider the even more general case of measurements based on pointer observables that are not necessarily sharp. It will be shown that the established proof strategy of the previous no-go theorems can be adapted so as to cover a certain class of unsharp pointer observables: those admitting definite values. Technically this corresponds to the case of a positive operator-valued (POV) measure Z on a σ -algebra Σ which is such that each effect $Z(X)$, $X \in \Sigma$, has eigenvalue 1. The corresponding eigenstates are those

¹Department of Mathematics, University of Hull, U.K.; e-mail: p.busch@maths.hull.ac.uk.

states in which the pointer has *definite values*. Note that this does not require the effect to be a projection. It will be shown that no unitary measurement exists in which the compound object-plus-apparatus system could always (i.e., for arbitrary initial object states) be in a mixture of states in which the extended pointer ($I \otimes Z$) would have definite values. The occurrence of such a mixture is a necessary condition for the *pointer objectification* (Busch *et al.*, 1996). The proof technique used here differs from that used by Shimony (1974) and Busch and Shimony (1996) in that a recent theorem due to Stein (1997) is applied. This provides an extension of his impossibility theorem and the insolubility theorem of Busch and Shimony (1996).

This result implies that the quantum measurement problem is not simply due to idealizations in which possible measurement inaccuracies are neglected: in using the general representation of observables as *POV* measures, all kinds of inaccuracy have been taken into account—to the extent they are still compatible with the idea of definite pointer values. The remaining potential loophole is furnished by the case of pointer observables which are *genuinely unsharp* in that they do not allow for pointer value definiteness. This opens up the challenge to make precise sense of the idea of *unsharp objectification*, which will be done here.

Apart from the possibility that no insolubility theorem might hold for genuinely unsharp pointers, the existing no-go theorems allow an exhaustive systematic overview of the possible modifications of quantum mechanics, or of its interpretations, that may be, and have been, undertaken to resolve (or dissolve) the measurement problem (Busch *et al.*, 1996).

2. NOTION OF MEASUREMENT IN QUANTUM MECHANICS

In the following I shall adopt the usual Hilbert space formulation of quantum mechanics, where observables and states are represented as, and identified with, certain positive operator-valued (POV) measures and density operators, respectively. These concepts are required to formulate the probability structure of the theory in its (probably) most general form. Then according to the minimal interpretation of quantum mechanics, the probability measures provided by the formalism give the probability distributions for measurement outcomes, and thus the expected experimental statistics.

This minimal notion of an observable—and of a measurement—is captured in the so-called *probability reproducibility* condition. The essential elements of a measurement are conveniently summarized in the concept of a *measurement scheme*, represented as a quadruple $\mathcal{M} := \langle \mathcal{H}_{\mathcal{A}}, \rho_{\mathcal{A}}, U, Z \rangle$, where $\mathcal{H}_{\mathcal{A}}$ denotes the Hilbert space of the measuring device (or probe) \mathcal{A} , Z the pointer observable of \mathcal{A} , i.e., a *POV* measure on some measurable space, (Ω, Σ) , $\rho_{\mathcal{A}}$ a fixed initial state of \mathcal{A} and U the unitary measurement

coupling serving to establish a correlation between the object system \mathcal{S} (with Hilbert space \mathcal{H}) and \mathcal{A} . Any measurement scheme \mathcal{M} fixes a unique observable of \mathcal{S} , that is, a POV measure E on (Ω, Σ) such that the following condition is fulfilled.

- **Probability Reproducibility Condition:**

$$\text{tr}[I \otimes Z(X)U\rho \otimes \rho_{\mathcal{A}}U^*] = \text{tr}[E(X)\rho] \tag{PR}$$

for all states ρ of \mathcal{S} and all outcome sets $X \in \Sigma$.

E is the observable measured by means of \mathcal{M} . Conversely, if an observable E of \mathcal{S} is given, then this condition determines which measurement schemes \mathcal{M} serve as measurements of E .

3. THE OBJECTIFICATION PROBLEM

The probability reproducibility condition specifies what it means that a measurement scheme serves to measure a certain observable. However, this condition does not exhaust the notion of measurement. In fact the reproduction of probabilities in the pointer statistics requires first of all that in each run of a measurement a pointer reading will occur; in other words: it is part of the notion of measurement that measurements do have definite outcomes. While the concept of a measurement scheme allows one to *describe* what happens to the object and apparatus when an outcome arises, quantum mechanics is facing severe difficulties to *explain* the occurrence of such outcomes. This problem arises if one starts with the interpretational idea that an observable has a definite value when the object system in question is in an eigenstate of that observable. If a probe system is coupled to that object, then probability reproducibility requires that the corresponding value is indicated with certainty by the pointer reading after the measurement interaction has ceased. In this way a definite value of the object observable leads deterministically to a definite value of the pointer observable. However, if the object is *not* in an eigenstate, the observable cannot be ascertained to have a definite value, and by the linearity of the unitary measurement coupling, the compound object-plus-probe system ends up in a state in which it cannot be ascertained, by appeal to the eigenstate–eigenvalue link, that the pointer has a definite value. This is the measurement problem, or the problem of the objectification of pointer values.

Resolutions to this problem are being sought by changing the rules of the game: either on the side of the formalism (introduction of classical observables, or modified dynamics), or on the interpretational side (hidden variables theories such as ‘Bohmian mechanics’ or various ‘no-collapse’

interpretations). Before embarking on such radical revisional programs, it seems fair to make sure that the measurement problem is not merely a consequence of overly idealized assumptions that would disappear in a more realistic account. It turns out, however, that the problem does persist even when measurements are allowed to be inaccurate and the measuring system is in a mixed rather than a pure state. The development of these arguments is reviewed in Busch and Shimony (1996), where an insolubility theorem is given that pertains to measurements of sharp and unsharp object observables. This result has recently been overtaken by Stein (1997), who showed that the objectification problem persists for arbitrary measurement schemes also when the measurement is not required to be applicable to *all* object preparations, but only to states in some subspace of the object's Hilbert space. Based on this result, a further step will now be taken that comprises the possibility of the pointer being an unsharp observable as well, as long as pointers can still assume definite values.

In order to give the precise statement of the insolubility theorem, let us consider a measurement scheme \mathcal{M} . The theorem is based on the following requirements as necessary conditions for the definiteness, or objectivity, of sharp values of the pointer Z in the postmeasurement state:

$$\rho'_{\mathcal{F}\mathcal{A}} \equiv U \rho_{\mathcal{F}} \otimes \rho_{\mathcal{A}} U^*$$

- **Pointer mixture condition:**

$$\rho'_{\mathcal{F}\mathcal{A}} = \sum I \otimes Z(x_i)^{1/2} \rho'_{\mathcal{F}\mathcal{A}} I \otimes Z(x_i)^{1/2} \equiv \sum \rho'_{\mathcal{F}\mathcal{A}}(X_i) \quad (\text{PM})$$

for some partition $\Omega = \cup X_i$ and all initial object states ρ .

- **Pointer value definiteness:**

$$\text{tr}[I \otimes Z(x_i) \rho'_{\mathcal{F}\mathcal{A}}(X_i)] = \text{tr}[\rho'_{\mathcal{F}\mathcal{A}}(X_i)] \quad (\text{PVD})$$

for all i and all initial object states ρ .

For a derivation of these conditions, see Busch *et al.* (1996). The first says that the postmeasurement state should be a mixture of pointer eigenstates, while the second requires that the final states conditional on reading a result in X_i are indeed eigenstates of the pointer for which X_i has probability one to occur again upon immediate repetition of the reading of the pointer observable Z .

Insolubility Theorem. If a measurement scheme \mathcal{M} fulfills (PM) and (PVD) for all object states ρ supported in some subspace \mathcal{H}_0 of \mathcal{H} , then the measured observable E according to (PR) is *trivial* with respect to all such

states; that is, $\text{tr}[E(X)\rho] = \lambda(X)$ for all $X \in \Sigma$, where λ is a state-independent probability measure on (Ω, Σ) . Hence if a measurement scheme is to lead to objective pointer values, it will yield no information at all about the object.

4. PROOF OF THE INSOLUBILITY THEOREM

We make use of the following lemma by Stein (1997), applying it very much in the same way as Stein himself did, but using our terminology and allowing for unsharp pointers.

Lemma. Let Q, R be bounded linear operators on $\mathcal{H}_{\mathcal{A}}$ and $\mathcal{H} \otimes \mathcal{H}_{\mathcal{A}}$, respectively. Let \mathcal{H}' be a vector subspace of \mathcal{H} . Assume that for all nonzero vectors $\varphi \in \mathcal{H}'$,

$$(P[\varphi] \otimes Q)R = R(P[\varphi] \otimes Q)$$

(Here $P[\varphi]$ denotes the projection onto the ray containing φ .) Then there exists a unique bounded linear operator $\tilde{\rho}_{\mathcal{A}}$ in $\mathcal{H}_{\mathcal{A}}$ such that

$$(P[\varphi] \otimes Q)R = P[\varphi] \otimes \tilde{\rho}_{\mathcal{A}} \quad \text{for all } \varphi \in \mathcal{H}_0$$

We apply this as follows: for any $\varphi \in \mathcal{H}_0$ we denote $\rho_{\mathcal{G}\mathcal{A}}(\varphi) := P[\varphi] \otimes \rho_{\mathcal{A}}$ and $\rho'_{\mathcal{G}\mathcal{A}}(\varphi) := U(P[\varphi] \otimes \rho_{\mathcal{A}})U^{-1}$. By assumption (PVD), any nonzero effect $Z(X)$ has eigenvalue 1. Let $Z(X_i)^{(1)}$ denote the corresponding spectral projection of the effect $Z(X_i)$. Then the assumption (PM) is equivalent to saying that each nonzero component state $\rho'_{\mathcal{G}\mathcal{A}}(X_i)$ is an eigenstate of $Z(X_i)^{(1)}$ associated with the eigenvalue 1, that is, $Z(X_i)^{(1)}\rho'_{\mathcal{G}\mathcal{A}}(X_i) = \rho'_{\mathcal{G}\mathcal{A}}(X_i)$ for all X_i of the given partition. Therefore (PM) implies that $\rho'_{\mathcal{G}\mathcal{A}}(\varphi)$ commutes with all $I \otimes Z(X_i)^{(1)}$, and thus also with all $I \otimes Z(X_i)$:

$$[I \otimes Z(X_i), \rho'_{\mathcal{G}\mathcal{A}}(\varphi)] = 0$$

We rewrite this as follows:

$$[U^{-1}(I \otimes Z(X_i))U, P[\varphi] \otimes \rho_{\mathcal{A}}] = 0$$

Now we make the following choices for the operators R, Q introduced in the Lemma: for each i , let $R_i = U^{-1}(I \otimes Z(X_i))U$ and $Q_i = \rho_{\mathcal{A}}$. Then by virtue of the Lemma there exists an operator $\tilde{\rho}_{\mathcal{A}}(X_i)$ such that

$$P[\varphi] \otimes \rho_{\mathcal{A}}U^{-1}(I \otimes Z(X_i))U = P[\varphi] \otimes \tilde{\rho}_{\mathcal{A}}(X_i)$$

Taking the trace yields the probabilities for the measured observable $E^{\mathcal{A}}$.

$$\text{tr}[P[\varphi]E^{\mathcal{A}}(X_i)] = \text{tr}[\tilde{\rho}_{\mathcal{A}}(X_i)]$$

As the operators $\tilde{\rho}_{\mathcal{A}}(X_i)$ are independent of φ , it follows that the measured observable is trivial with respect to states from the subspace \mathcal{H}_0 . This completes the proof.

5. UNSHARP OBJECTIFICATION

The residual question left open by the above result is whether the conclusion of ‘no information gain’ remains valid if the assumption (PVD) of definite pointer values is dropped. That is, one would only require a modified form of (PM) to hold: the final object-plus-apparatus state should be a mixture of states,

$$\rho'_{\mathcal{I}S} = \sum_i \rho'_{\mathcal{I}S}(X_i)$$

in which the pointer is *unsharply real*. By this we mean that the component states should be ‘near-eigenstates’ of $I \otimes Z(X_i)$ in the sense that they give probabilities close to one for the corresponding X_i . If in addition it can be ascertained that the above mixture admits an ignorance interpretation, then it shall be said that *unsharp objectification* has taken place.

Unsharp objectification, as explained here, would be a rather natural option if the pointer observables available in realistic experiments were genuinely unsharp (so that they would not allow for probabilities equal to one). One can argue that pointers, being macroscopic quantities, are in fact of that kind. Some of the arguments supporting this conclusion are detailed in Busch *et al.* (1995, 1996). Unsharp pointer readings correspond to a situation where the pointer states associated with different values are not (strictly) orthogonal. Thus one cannot claim with certainty that the reading one means to have taken is reproducible on a ‘second look’ at the pointer. For macroscopic quantities, however, the potential error will be practically negligible, as it can be extremely small compared to the scale of the reading.

Yet I would conjecture that unsharp objectification cannot be achieved either. Once this would have been established, one could safely conclude that the only way out lies in some of the mentioned modifications either of the formalism or the interpretation of quantum mechanics. Nevertheless it seems worthwhile to pursue the notion of unsharp pointers, as it may contribute to resolving some problems these alternative approaches are still facing, such as the so-called tail problem that arises in the case of the (continuous) spontaneous collapse models.

ACKNOWLEDGMENTS

For reasons which those who were present at my talk at the Quantum Structures 1996 Meeting will understand, I felt “forced” by the Chairman’s presence to finish my talk five minutes early. (“A *watched speaker* never ends late.”) I am grateful this happened (the responsibility for which is all mine), as it happily induced me to elaborate (at least to my satisfaction) those parts of my manuscript that I “had to” skip in my talk.

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